Physics Notes

BY

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Class:10+2
Unit: III
Topic: Magnetic Field due to current

__SYLLABUS__: UNIT-III-*A,B*

Concept of magnetic field, Oersted's experiment, Biot-Savart law, magnetic field due to an infinitely long current carrying straight wire and a circular loop; Ampere's circuit law and its applications to straight and toroidal solenoids; Force on a moving charge in uniform magnetic and electric fields, Cyclotron; Force on current – carrying conductor in a uniform magnetic field. Forces between two parallel current- carrying conductors-definition of ampere; Torque experienced by a current loop in a uniform magnetic field, moving coil galvanometer- its current sensitivity and conversion to ammeter and voltmeter.

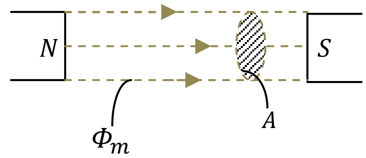
Current loop as a magnetic dipole and its magnetic dipole moment; Magnetic dipole moment of a revolving electron; Magnetic field intensity due to magnetic dipole (bar magnet) along the axis and perpendicular to the axis; Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; Bar magnet as an equivalent solenoid, Magnetic field lines' Earth's magnetic field and magnetic elements; Para-dia and ferro-magnetic substances with examples, Electromagnets and permanent magnets.

Q.1. Define magnetic field strength B? Units? Dimensions?

Ans. B → Magnetic flux per unit Area

$$B = \frac{\Phi_m}{A}$$

Where Φ_m → magnetic flux
A → Area



S.I. Unit: $B = \frac{Magnetic\ Flux}{Area}$

$= \frac{weber}{m^2}$ (wb/m^2)

1 tesla $= \frac{1\ wb}{m^2}$

Dimensions: $\vec{f} = q(\vec{V} \times \vec{B})$

$f = q\ V\ B$

Dimensions of, B $= \frac{MLT^{-2}}{AT(LT^{-1})}$

$B = \frac{f}{q.V}$

$(B) = [M^1L^0T^{-2}A^{-1}]$

$\vec{f} = q(\vec{V} \times \vec{B})$

$f = q\ V\ B$ sin 90

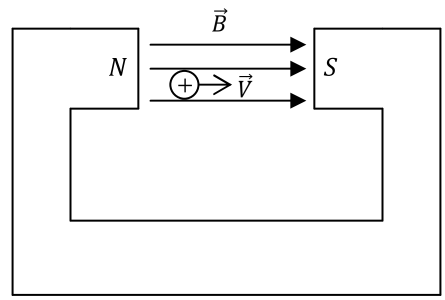
$\frac{f}{q\ V\ \sin 90} = B$

Magnetic field strength can be defined as Force acting on unit charge moving with unit velocity perpendicular to magnetic field.

Q.2. Draw diagrams of various cases of $\vec{f} = q(\vec{V} \times \vec{B})$.

Ans. $\vec{f} = q(\vec{V} \times \vec{B})$

Case-I. $\vec{V} \,||\, \vec{B}, \theta = 0$



$\vec{f} = q(\vec{V} \times \vec{B})$

$\vec{f} = qVB \sin 0$

$f = 0$

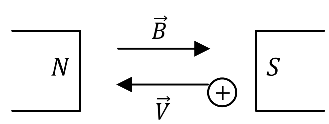
Whenever charge moves parallel to magnetic field, force acting on charge is *ZERO*

Case-II. $\Theta = 180^0$

$f = q(\vec{V} \times \vec{B})$

$f = q\,V\,B\,\text{Sin}\,180$

$f = 0$



Whenever charge moves antiparallel to magnetic field, force acting on charge is *ZERO*

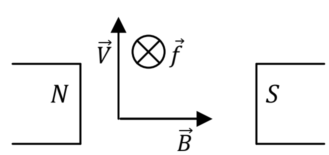
Case-III. $\Theta = 90^0$

$f = q(\vec{V} \times \vec{B})$

$f = q\,V\,B\,\text{Sin}\,90$

$f = q\,V\,B$

$f_{max} = q.V.B$



Force is max whenever charge moves ⊥ to magnetic field

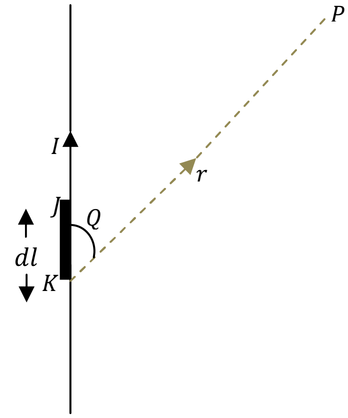
Direction:-

$\vec{f}$ is normal to $\vec{V}$ and $\vec{B}$

$\vec{f}$ is inwards as per right hand screw rule

Q.3. Explain Biot – Savarts Law?

Ans. Current Element → I $\vec{dl}$ (S.I. unit of current element is amp-metre)



dB → small magnetic field at *P* due to *I.dl*

$dB \propto Idl$ ——— (1)

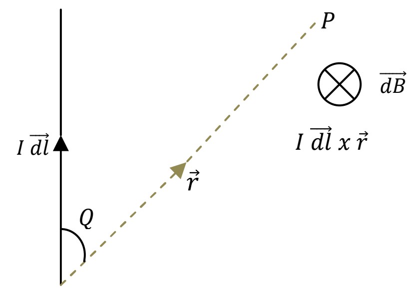
$dB \propto \frac{1}{r^2}$ ——— (2)

Combining (1) and (2)

$$dB \propto \frac{I\,dl}{r^2}$$

$$\overrightarrow{dB} = (Constant)\,\frac{I\,\vec{dl}\times\vec{r}}{r^3}$$

$$\boxed{\overrightarrow{\boldsymbol{dB}} = \left(\frac{\mu_0}{4\pi}\right)\frac{\boldsymbol{I\,\vec{dl}\times\vec{r}}}{\boldsymbol{r^3}}}$$



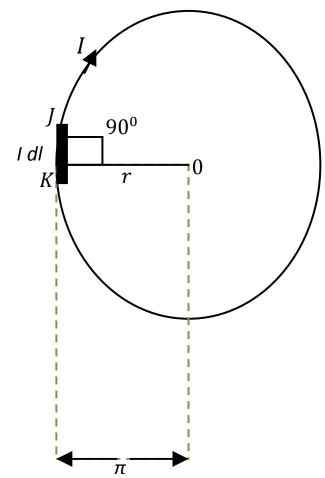
$\mu_0 = 4\pi \times 10^{-7}$ (S.I. unit), is permeability of free space

Direction of $I\,\vec{dl}\times\vec{r}$ is inwards as per right hand screw rule

Q.4. Use Biot Savarts Law to find field at the centre of ring of radius *r* carrying current *I*.

Ans. Step 1.

Apply Biots Savarts Law for small elements JK



$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl\,r\,\sin 90^0}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\,dl(1)}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{r^2}$$

Step 2.

Total field at 0 i.e. at centre of ring

Integrate both sides

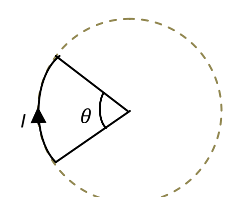
$$\int dB = \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0 . I}{4\pi r^2} . \int dl$$

$$= \frac{\mu_0 . I\, 2\pi r}{4\pi r^2}$$

$$B = \frac{\mu_0 . I}{2 . r}$$

For arc of angle θ



$$B = \frac{\mu_0 . I}{4\pi r} . \left(\frac{\theta}{2\pi}\right)$$

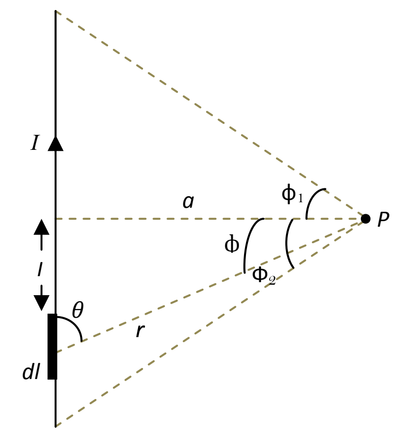
For full circle $\theta = 2\pi$

$$B_{circle} = \frac{\mu_0 . I}{4\pi r} . (2\pi)$$

$$B_{circle} = \frac{\mu_0 . I}{2 . r}$$

Q.5. Use Biot Savarts Law to find field due to current carrying conductor of finite length?

Ans. $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \vec{r}}{r^3}$



Step 1.

Field due to $I\,\vec{dl}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I.dl\, r.\sin(\theta)}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I.dl.r.\sin\theta}{r^3}$$

Step 2.

Integrating both sides

$$dB = \int \frac{\mu_0}{4\pi} \frac{I\,dl\,r\sin\theta}{r^3}$$

$$B = \frac{\mu_{0.I}}{4\pi} \int \frac{I\,dl\,r\sin\theta}{r^3}$$

$$B = \frac{\mu_{0.I}}{4\pi} \int \frac{dl\,\cos\phi}{r^2}$$

$\Phi + \Phi = 90^\circ$

$\sin\theta = \cos\Phi$

$$= \frac{\mu_{0.I}}{4\pi} \int \frac{a\sec^2\phi.\,d\phi.\,\cos\phi}{a^2\sec^2\phi}$$

$$= \frac{\mu_{0.I}}{4\pi} \int \cos\phi\, d\phi$$

$$= \frac{\mu_{0.I}}{4\pi} \left| \sin\phi \right|_{\phi = \phi_2}^{\phi = \phi_1}$$

$$\boxed{B = \frac{\mu_{0.I}}{4\pi a}(\sin\phi_1 + \sin\phi_2)}$$

<u>Mathematical</u>

$\frac{l}{a} = \tan\phi$

$l = a\tan\phi$

$$\boxed{dl = a\sec^2\phi.d\phi}$$

Also

$\frac{a}{r} = \cos\phi$

$r = a\sec\phi$

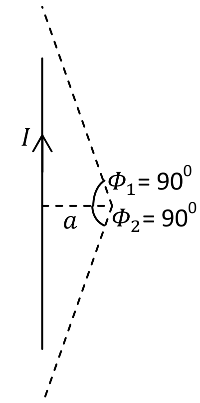
Special Cases:

Case-I: infinite conductor (at centre)

$\Phi_1 = \phi_2 = 90^0$

$$B = \frac{\mu_{0.I}}{4\pi a}(\sin 90^0 + \sin 90^0)$$

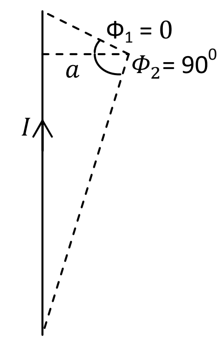
$$\boxed{B = \frac{\mu_{0.I}}{4\pi a}}$$



Case-II: Infinite conductor (at end)

$$B = \frac{\mu_{0.I}}{4\pi a}(\sin 0^0 + \sin 90^0) \quad \boxed{\Phi_1 = 0^0, \Phi_2 = 90^0}$$

$$\boxed{B = \frac{2\mu_{0.I}}{4\pi a}}$$

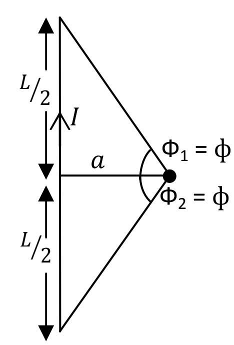


Case-III:

Length of the conductor is finite say L and point P lies on right bisector of conductor, then $\Phi_1 = \phi_2 = \phi$

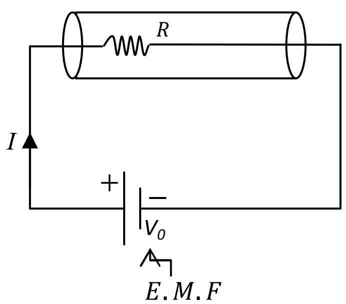
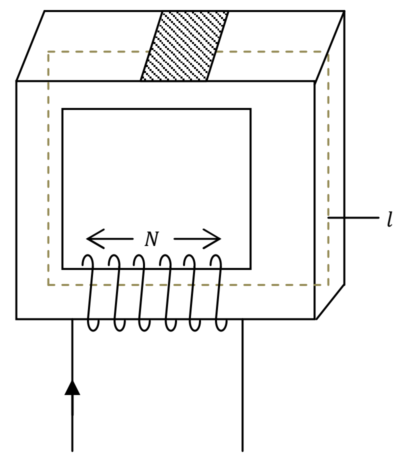
$$\text{Sin}\phi = \frac{L/2}{\sqrt{a^2 + \left(\frac{L}{2}\right)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

$$\boxed{B = \frac{\mu_{0.I}}{4\pi a}(\sin\phi + \sin\phi)}$$



Q.6. Compare Electric and Magnetic Circuit?

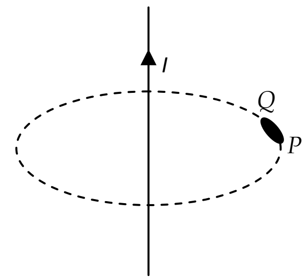
Ans.

Electric Circuit	Magnetic Circuit
1. What causes current in the Electric Circuit?	1. What causes magnetic flux in magnetic circuit?
Ans. *Emf* (Electro motive force)	Ans. *Mmf* (Magneto motive force)
	
2. Current, I	2. Magnetic flux, ϕ
3. Resistance, $R = \dfrac{l}{\sigma . A}$ (σ → Conductivity of material)	3. Reluctance, R_e {opposition to flow of magnetic flux} $R_e = \dfrac{l}{\mu . A}$ (μ → Permeability) Iron has high permeability wood has low permeability $\mu = \mu_0 . \mu_r$ (μ_r → [1,2,3,4,.....] is relative permeability) μ_0 → permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ in SI units.
4. E, Electric field intensity = $E = \dfrac{V}{l} = \dfrac{e.m.f}{length}$	4. H, Magnetising field intensity = $H = \dfrac{m.m.f}{l} = \dfrac{NI}{l}$ N → number of turns l → length of magnetic circuit

Electric Circuit	**Magnetic Circuit**
5. $J = \frac{I}{A}$ Current density	5. Magnetic flux density $\vec{B} = \frac{manetic\ flux}{Area}$ $= \frac{\phi}{A}$
6. Ohm's Law $V = IR$ $I = \frac{V}{R}$ $= \frac{e.m.f}{R}$	6. Flux $\phi = \frac{m.m.f}{R_e}$ $\phi = \frac{N.I}{R_e}$ $\Phi = \frac{N.I}{R_e}$ $= \frac{N.I}{\left(\frac{l}{\mu A}\right)}$ $\frac{\phi}{A} = \frac{N.I\mu}{l}$ $B = \frac{\mu(N.I)}{l}$ [H = $\frac{N.I}{l}$ is *m.m.f.* per unit length] $B = \mu H$ Magnetic flux density depends upon material but *H* does not depend on material.

Q.7. State and Explain Ampere – Circuit Law?

Ans. $m.m.f$ in part PQ is = H.dl



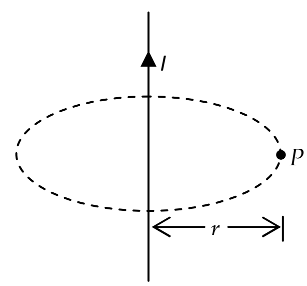
" Total $m.m.f$ in closed loop"

$$= \oint H.dI$$

"Total $m.m.f$ in a closed loop is equal to total current contained".

$$\boxed{\oint H.dI = I}$$

Q.8. Use Ampere-Circuit Law to find magnetic field due to current carrying infinite conductor.



Ans. Step 1.

Apply ampere circuit law in loop,
Total *m.m.f.* in closed loop = Total current contained

i.e. $\oint H.dI = I$

Step 2.

Take H → constant

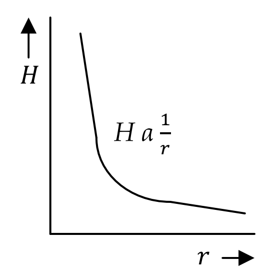
$H.\oint dl = I$

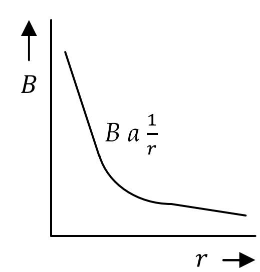
$H.2\pi r = I$ (As $\oint dl = 2\pi r$)

$$\boxed{H = \frac{I}{2\pi r}}$$

$$\boxed{B = \mu_0 H}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$



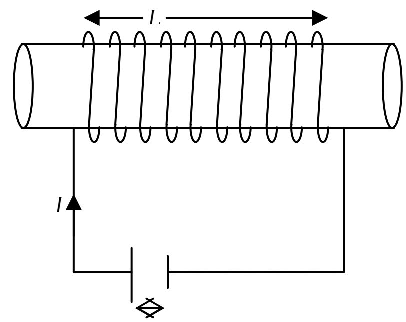


Q.9. Find magnetic field at centre of infinite solenoid carrying current *I*.

Ans. N → total no. of lines

L → total length

n → $\frac{N}{L}$ = $\frac{no.of\ turns}{length}$

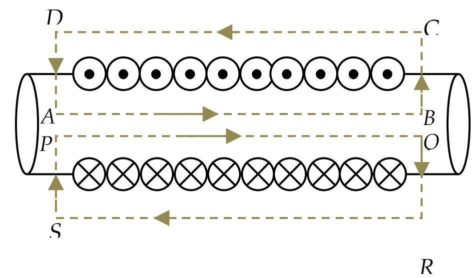


<u>Proof</u>:

<u>Step 1</u>.

Make ampere circuit loop

Apply ampere circuit law for loop ABCD



$\oint H.dl = N.I$

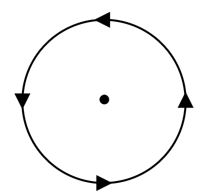
ABCDA

<u>Step 2</u>.

Ampere circuit loop sub parts

$\int_{AB} H.dl + \int_{BC} H.dl + \int_{CD} H.dl + \int_{DA} H.dl = N.I.$

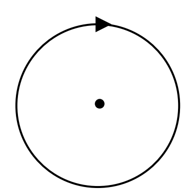
$\int_{AB} H.dl + 0 + \int_{CD} H.dl + 0 = N.I.$



$2\int H.dl \quad = NI$

$2H\int dl \quad = NI$

$2HL \quad = NI$



$H \quad = \frac{N.I}{2.L}$

$H_{total} \quad = 2.\left(\frac{N.I}{2.L}\right) = \left(\frac{N}{L}\right).I$

<u>Step 3</u>.

$H_{total} \quad = nI$

<u>Step 4</u>.

$B \quad = \mu_0 H_{total}$ (for air)

$B \quad = \mu_0 nI$ (for air solenoid)

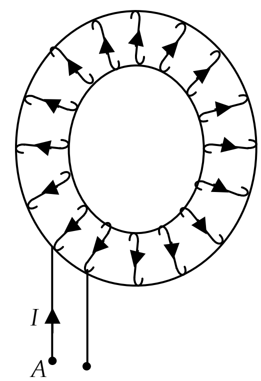
$B \quad = \mu_0.\mu_r.nI$

↓ Relative permeability material

Q.10. Use ampere circuit law to find magnetic field in and around a toroid?

Ans. Case I.

Inside Toroid (iron part)



Ampere Circuit Law

$\oint H.dl = NI$

$H.\oint dl = NI$

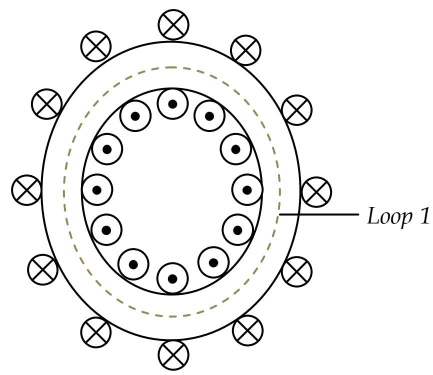
$H(2\pi r) = NI$

$H = \left(\frac{N}{2\pi r}\right).I$

$H = nI$

$B = \mu H$

$$\boxed{B = \mu nI}$$



Case-II. (Loop 2)

Outside Toroid

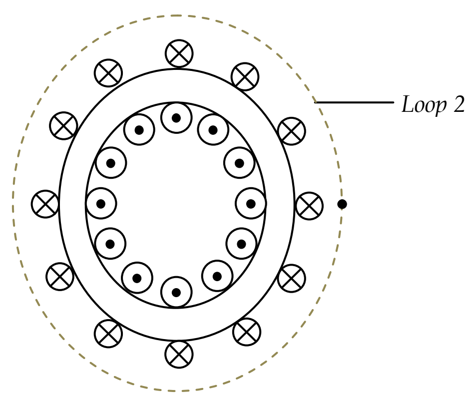
$\oint H.dl = N(+I) + N(-I)$

$H.\oint dl = 0$

$H.2\pi r = 0$

$H = 0$

$$\boxed{B = 0}$$



Case-III. (Loop 3)

$\oint H.dl = 0$

$H.2\pi r = 0$

$H = 0$

$$\boxed{B = 0}$$